Phase 10 – Part 3  
Classification of Stable vs. Unstable ψ Modes

Goal  
In this part, I aim to classify the behavior of ψ perturbations into stable, unstable, and marginal regimes. This builds directly on the dispersion relation derived in Part 2:

Plain text:  
ω² = C₀ k²

The classification depends on the sign and structure of the curvature factor , which encodes the Laplacian of space + current².

Recap  
Perturbation ansatz:

Plain text:  
φ(x, t) = exp(i(k·x − ω t))

Dispersion relation:

* : propagating oscillations.
* : exponential instability.
* : neutral equilibrium.

Local vs. Global Classification  
The curvature factor is generally spatially dependent:

Plain text:  
C(x) = ∇²(space(x) + current(x)²)

Thus, perturbations may experience different regimes depending on the local sign of .

1. Stable Modes  
   Condition:

Plain text:  
C(x) > 0

Implication:

* Oscillatory solutions with real frequencies.
* Stable ψ ripples that propagate like sound waves.
* No energy divergence.

Desert analogy: ripples form on firm sand where wind + dune balance maintains structure.

1. Unstable Modes  
   Condition:

Plain text:  
C(x) < 0

Implication:

* Imaginary frequency → exponential growth/decay.
* Small perturbations amplify, forming nonlinear dune collapse.
* Indicates ψ well is structurally unstable in that region.

Desert analogy: shifting sands collapse, ripples turn into growing dunes, chaotic motion emerges.

1. Marginal Modes  
   Condition:

Plain text:  
C(x) = 0

Implication:

* .
* Perturbations neither propagate nor grow.
* ψ is “floppy,” like a neutrally balanced flat dune.

Desert analogy: the desert floor is perfectly flat, disturbances simply sit without evolving.

Mixed Domains  
Since ψ-gravity operates over extended domains, regions may differ in curvature sign. Perturbations in such systems may:

* Oscillate in regions with .
* Amplify when entering zones.
* Freeze in neutral areas.

This suggests mode conversion: stable waves can destabilize when crossing into negative-curvature zones, analogous to wave refraction in inhomogeneous media.

Stability Diagram  
Define regimes in a plane:

* Stable region: .
* Unstable region: .
* Neutral line: .

Numerical Illustration

# simulations/phase10\_part3\_classification.py  
import numpy as np  
import matplotlib.pyplot as plt  
  
# Define curvature range  
C\_values = np.linspace(-2, 2, 400)  
k = 1.0 # fixed wavenumber  
  
# Dispersion relation  
omega2 = C\_values \* k\*\*2  
  
# Classify regimes  
stable = omega2 > 0  
unstable = omega2 < 0  
neutral = omega2 == 0  
  
plt.axhline(0, linestyle='--')  
plt.plot(C\_values, omega2, label="ω² vs. C")  
plt.fill\_between(C\_values, omega2, 0, where=stable, alpha=0.3, label="Stable")  
plt.fill\_between(C\_values, omega2, 0, where=unstable, alpha=0.3, label="Unstable")  
plt.xlabel("Curvature factor C₀")  
plt.ylabel("ω²")  
plt.title("Phase 10 Part 3: ψ Mode Stability Classification")  
plt.legend()  
plt.show()